Math 1050-003 Midterm 1
Spring 2016

Name: Solutions UNID:

- **SHOW ALL WORK**. No points will be given for answers without justification.
- The more you show you understand the material, the more points you will receive.
- If you are using a formula to complete a problem, make sure you **write down the formula** first.
- **Simplify** your answer as much as you can without a calculator unless directions specify otherwise.
- Make sure your handwriting is legible.
- Put your **FINAL** answer in the space provided on the exam so that it is easy to locate.
- Calculators, homework, notes, phones, or any other external aid are **NOT** permitted.
- Absolutely **no talking** is permitted during the midterm.
- Be sure to work carefully and **check your answers** before turning in your exam. Once an exam has been turned in, you will not be allowed to alter your exam for any reason.
- Good Luck!!

1. **(8 points)** Determine whether the following statements are true or false and explain why. **No points will be given if there is no explanation.**

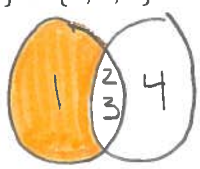
   (a) $3 \in (-1, 3]$

   (1a) True. 3 is an object in the interval $(-1,3]$

   (b) $\mathbb{N} \subseteq \mathbb{Q} - \{1\}$

   (1b) False. $1 \in \mathbb{N}$ but $1 \notin \mathbb{Q} - \{1\}$

   (c) $\{1, 2, 3\} - \{2, 3, 4\} = \{1, 4\}$



   (1c) False. $\{1,2,3\} - \{2,3,4\} = \{1\}$

   (d) Suppose $f : \mathbb{Z} \to \mathbb{R}$ where $f(x) = \dfrac{1}{x^2}$
   Then $f(\frac{1}{2}) = 4$

   (1d) False. $f(\frac{1}{2})$ DNE because $\frac{1}{2} \notin \mathbb{Z}$

2. **(6 points)** State if the sequence is arithmetic, geometric, or neither

   (a) $25, 10, 4, \ldots$

   $25r = 10$
   $r = \frac{10}{25} = \frac{\cancel{5}(2)}{\cancel{5}(5)} = \frac{2}{5}$

   (2a) Geometric

   (b) $1, 2, 1, 2, \ldots$

   (2b) Neither

   (c) $-9, -3, 3, \ldots$

   $-9 + d = -3$
   $d = 6$

   (2c) Arithmetic

3. **(3 points)** Given the sequence defined by $a_n = \frac{n^2-3}{n+5}$, find a_6

$$a_6 = \frac{(6)^2 - 3}{6+5} = \frac{36-3}{11} = \frac{33}{11} = 3$$

(3) $a_6 = 3$

4. **(3 points)** Find the 201^{st} term for the following sequence: $-5, 10, -20, 40, \ldots$

Geometric, $r = -2$

$a_k = a_1 r^{k-1}$

$a_{201} = (-5)(-2)^{200}$

(4) $a_{201} = (-5)(-2)^{200}$

5. **(4 points)** Calculate $\sum_{i=1}^{4}(2i-3)^2$

$= (2(1)-3)^2 + (2(2)-3)^2 + (2(3)-3)^2 + (2(4)-3)^2$
$= (2-3)^2 + (4-3)^2 + (6-3)^2 + (8-3)^2$
$= (-1)^2 + (1)^2 + (3)^2 + (5)^2$
$= 1 + 1 + 9 + 25$
$= 36$

(5) 36

6. **(4 points)** Find the indicated sum: $\sum_{i=1}^{\infty} 6\left(\frac{1}{3}\right)^{i-1}$

$= 6(\frac{1}{3})^0 + 6(\frac{1}{3})^1 + 6(\frac{1}{3})^2 + \ldots$
Geometric, $a_1 = 6$, $r = \frac{1}{3}$
$S = \frac{a_1}{1-r} = \frac{6}{1-\frac{1}{3}} = \frac{6}{\frac{2}{3}} \frac{(\frac{3}{2})}{(\frac{3}{2})} = \frac{18}{2} = 9$

(6) 9

7. **(4 points)** Find the sum of the first 101 terms of the following sequence: $3, 7, 11, 15, \ldots$
Express your answer as a product of integers.

Arithmetic, $d = 4$
$S = \frac{k}{2}(a_1 + a_k) = \frac{101}{2}(3 + a_{101}) = \frac{101}{2}(406) = 101(203)$
$a_{101} = 3 + 100(4) = 403$

(7) 101(203)

8. **(4 points)** Find the infinite sum: $\frac{1}{3} + 1 + 3 + 9 + \ldots$

Geometric, $r = 3$

(8) The sum diverges because $r > 1$

9. **(4 points)** To buy a new car, a customer can choose one of 2 body styles, one of 5 colors, and one of 3 models. Determine the number of possible new cars that a customer can choose from. **Simplify your answer completely so that you are left with an integer.**

Choice	Body	Color	Model
#options	2	5	3

$= 2(5)(3) = 30$

(9) 30

10. **(4 points)** If 20 people work in an office and 4 are selected to go to a conference how many different selections are possible? **You do not need to simply completely, but make sure any factorials(!) in your answer have been expanded.**

order doesn't matter

$$\binom{20}{4} = \frac{20!}{4!(20-4)!} = \frac{20!}{4!\,16!} = \frac{20(19)(18)(17)(\cancel{16!})}{4(3)(2)(1)(\cancel{16!})}$$

(10) $\frac{20(19)(18)(17)}{4(3)(2)(1)}$

11. **(4 points)** How many ways can you arrange the letters PANTHERS if you use all the letters? **You do not need to simply.**

order matters

$8!$

(11) $8!$

12. **(4 points)** Suppose an organization consists of 12 members. The organization is giving a presentation in which 3 members have to give speeches, one after another, and no two speeches are given by the same person. How many different line-ups are possible? **You do not need to simply completely, but make sure any factorials(!) in your answer have been expanded.**

order matters

Way 1:

Choice	1st	2nd	3rd
#options	12	11	10

$= 12(11)(10)$

Way 2:

$$\frac{12!}{(12-3)!} = \frac{12(11)(10)(\cancel{9!})}{\cancel{9!}}$$

(12) $12(11)(10)$

13. **(8 points)** Answer the following questions:

(a) Write out the first 5 rows of Pascal's triangle (n=0 to n=4) where the binomial coefficients are written as integers.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

(b) Use the Binomial Theorem and Pascal's triangle to expand $(x+y)^4$

$$(x+y)^4 = \sum_{i=0}^{4} \binom{4}{i} x^{4-i} y^i$$
$$= \binom{4}{0}x^4 + \binom{4}{1}x^3y + \binom{4}{2}x^2y^2 + \binom{4}{3}xy^3 + \binom{4}{4}y^4$$

(13b) $x^4+4x^3y+6x^2y^2+4xy^3+y^4$

(c) Use your results from part (b) to expand $(2x-1)^4$. Simplify completely.

$$(2x)^4 + 4(2x)^3(-1) + 6(2x)^2(-1)^2 + 4(2x)(-1)^3 + (-1)^4$$
$$= 16x^4 - 4(8)x^3 + 6(4)x^2 - 8x + 1$$

(13c) $16x^4-32x^3+24x^2-8x+1$

14. **(4 points)** State the implied domain of the following functions:

(a) $f(x) = x^2 - 2x + 4$ (14a) $\mathbb{R}$

(b) $g(x) = \dfrac{3x+1}{2x-1}$

$2x-1=0$
$2x=1$
$x=\frac{1}{2}$

(14b) $\mathbb{R} - \{\frac{1}{2}\}$
OR $(-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$

15. **(6 points)** If $f(x) = x^2 + 5$ and $g(x) = 2x + 1$ determine the following. **You do not need to simplify**.

(a) $f \circ g(x)$

$f(g(x)) = f(2x+1) = (2x+1)^2+5$

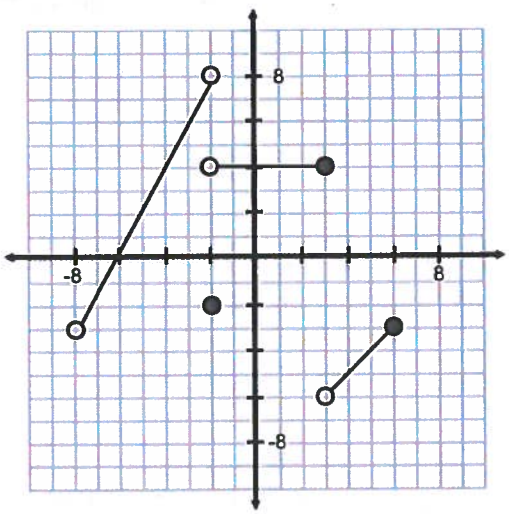
(15a) $(2x+1)^2+5$

(b) $g \circ f(x)$

$g(f(x)) = g(x^2+5) = 2(x^2+5)+1$

(15b) $2(x^2+5)+1$

16. **(10 points)** Below is the graph of a function $f(x)$. Use the graph to answer the following questions:



(a) What is $f(-2)$?

(16a) −2

(b) What is the domain of $f(x)$?

(16b) (−8, 6]

(c) What is the range of $f(x)$?

(16c) (−6, 8)

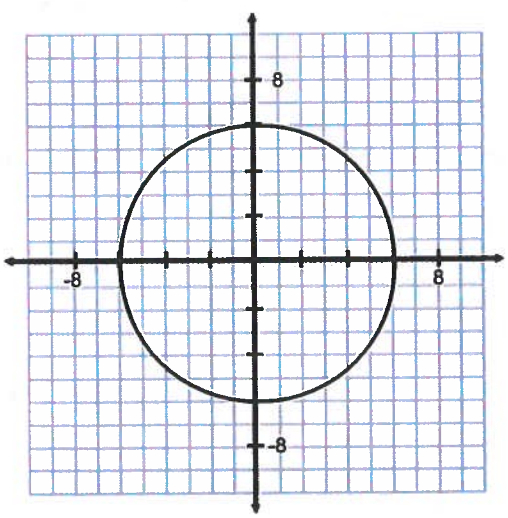
(d) State all x-intercepts as ordered pairs.

(16d) (−6, 0)

(e) State all y-intercepts as ordered pairs.

(16e) (0, 4)

17. **(2 points)** Is the picture below the graph of a function? Why or why not?

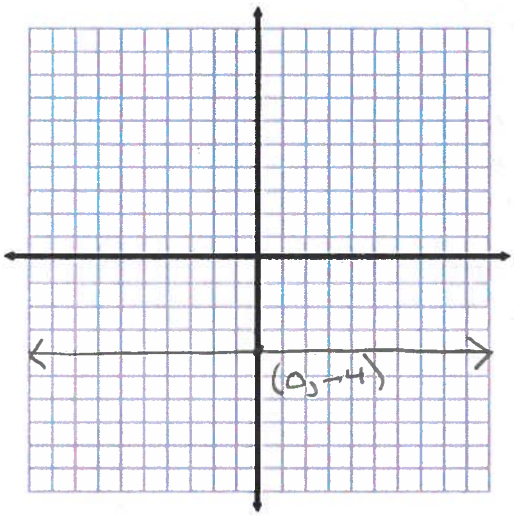


(17) No, it does not pass the vertical line test, which means that there is more than one output (y-value) for each unique input (x-value)

18. **(8 points)** Sketch the graph and state the domain (D) and range (R) for each of the following. Be sure to **label at least one point** on the graph.

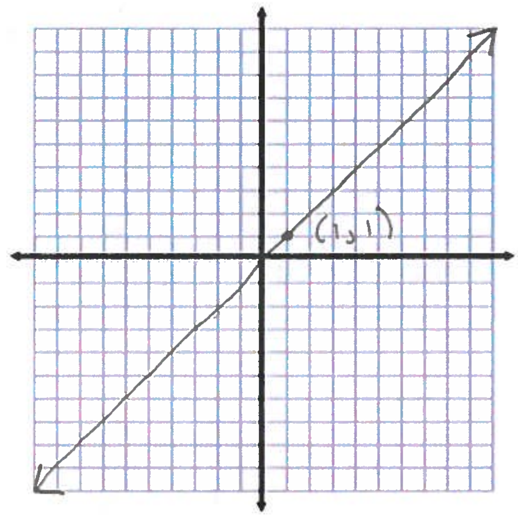
(a) $f(x) = -4$

D = $\mathbb{R}$ R = $\{-4\}$



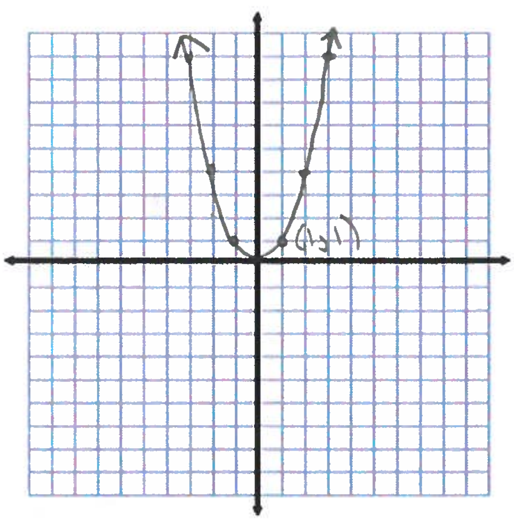
(b) $f(x) = x$

D = $\mathbb{R}$ R = $\mathbb{R}$



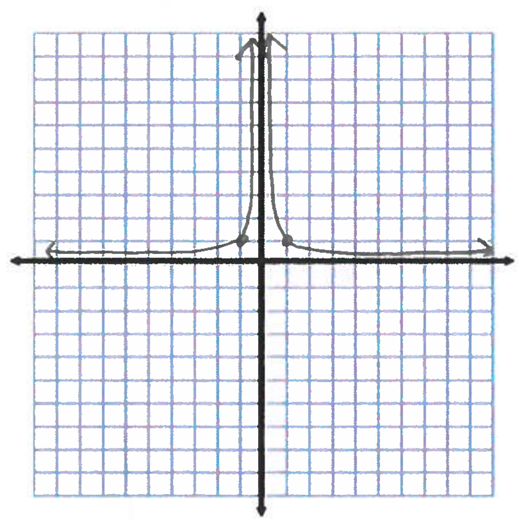
(c) $f(x) = x^2$

D = $\mathbb{R}$ R = $[0, \infty)$

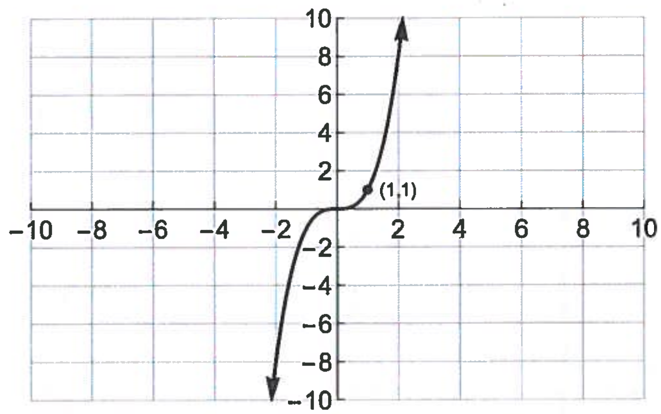


(d) $f(x) = \frac{1}{x^2}$

D = $\mathbb{R} - \{0\}$ R = $(0, \infty)$

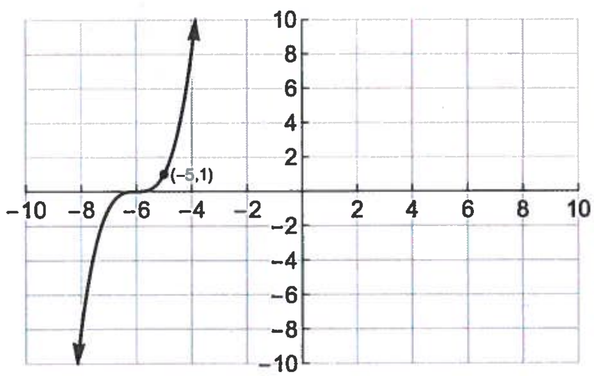


19. **(4 points)** Given the graph of the function $f(x) = x^3$ below,



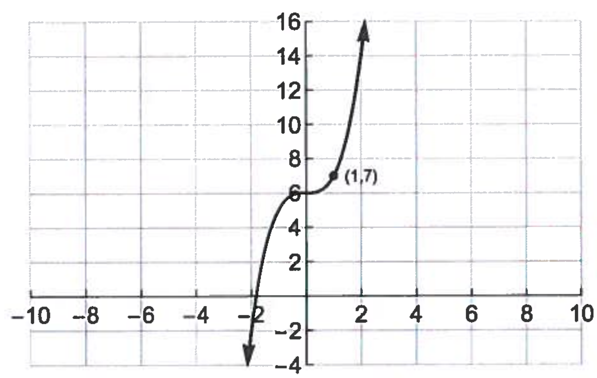
Label each of the figures below with one of the following choices:
$f(x+6)$, $f(x-6)$, $f(x)+6$, $f(x)-6$, $6f(x)$, $\frac{1}{6}f(x)$, $f(6x)$, $f(\frac{1}{6}x)$, $f(-x)$.

(a)



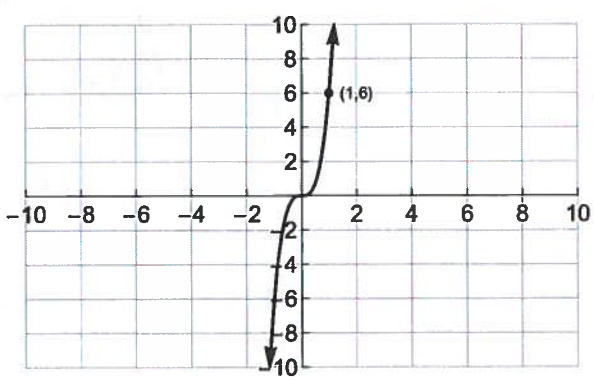
(19a) $f(x+6)$

(b)



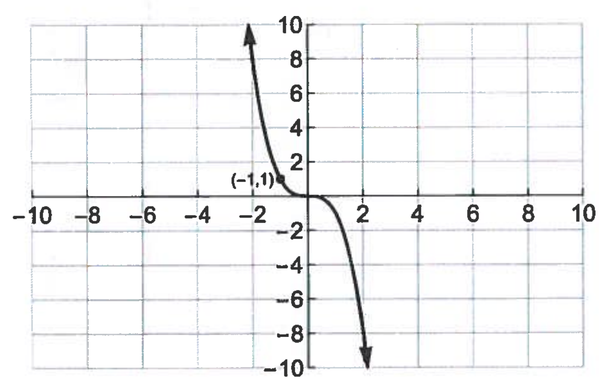
(19b) $f(x)+6$

(c)



(19c) $6f(x)$

(d)



(19d) $f(-x)$

20. **(6 points)** Use graph transformations to complete the following for the transformed function:

$$f(x) = (x+2)^2 - 1$$

(a) Define the original function and explain each step of the transformation in words.

Original Function: x^2

① Shift left by 2: $(x+2)^2$

② Shift down by 1: $(x+2)^2 - 1$

(b) Describe what happens to the point $(0, 0)$ by stating the new coordinates after each step of the transformation you described in (a).

$(0, 0)$

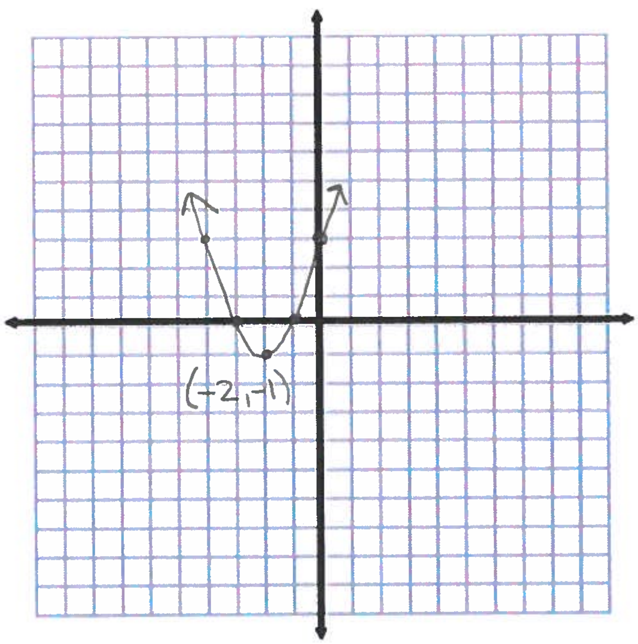
↓

① $(-2, 0)$

↓

② $(-2, -1)$

(c) Sketch $f(x) = (x+2)^2 - 1$. Be sure to **label at least one point** on the graph.



x	$f(x)$
0	3
-1	0
-2	-1
-3	0
-4	3